Dr. K. Karuppasamy

www.drkk.in

Yahoo Answers 21-12-2014

Problem : On a construction site gravel is delivered and poured into a conical pile. The diameter and height of the cone of gravel are changing in a way that the diameter is always three times the height. If the delivery truck is set for the gravel at a constant rate of 3 ft.³ permit how fast is the radius of the Pio changing when the height is 4 feet?

Solution: Let r be the radius, be height and V be the volume of the cone of gravel at time t sec.

It is given that, diameter = 3 * height and dV/dt = 3 ft^3/sec.

⇨ 2r = 3h and $\frac{dr}{dt} = \frac{3}{2}\frac{dh}{dt}$ ----------(1)

We have at any time t, $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h = \frac{3}{4}\pi h^3$

⇨ $\frac{dV}{dt} = \frac{9}{4}\pi h^2 \frac{dh}{dt}$

⇨ $\frac{dh}{dt} = \frac{4}{9\pi h^2}\frac{dV}{dt}$

When h = 4, $\frac{dh}{dt} = \frac{4}{9\pi 4^2} * 3 = \frac{1}{12\pi}$

When h = 4, (1) becomes, $\frac{dr}{dt} = \frac{3}{2} * \frac{1}{12\pi} = \frac{1}{8\pi}$

Hence radius of the Pio is changing at the rate of $\frac{1}{8\pi}$ ft/sec.