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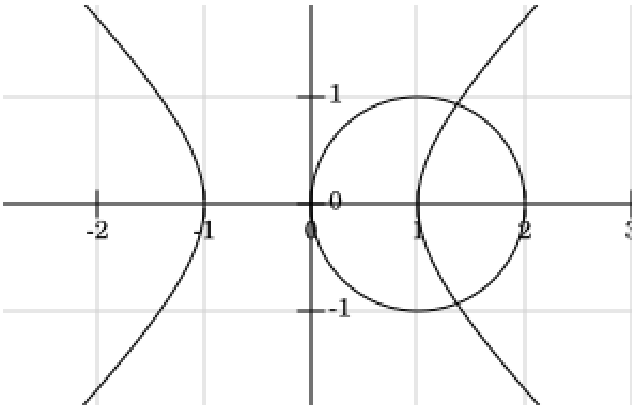
Problem: Using double integration, find the area bounded by the curves $x^2-y^2 = 1$ and $x^2+y^2-2x = 0$.

Solution:



Given $x^2-y^2 = 1$ ------(1)

and $x^2+y^2-2x = 0$ ------(2)

(1) is a hyperbola and (2) is a unit circle with center at (1, 0).

Solving (1) and (2), we get

$((1+\sqrt{3})/2, \sqrt{\sqrt{3}/2})$ and $((1+\sqrt{3})/2, -\sqrt{\sqrt{3}/2})$.

Since (1) and (2) are symmetric about x – axis.

Required area = $2\int_0^{\sqrt{\sqrt{3}/2}} \int_{\sqrt{1+y^2}}^{1+\sqrt{1-y^2}} dxdy$

$$= 2\int_0^{\sqrt{\sqrt{3}/2}} \left[1+\sqrt{1-y^2}-\sqrt{1+y^2}\right]dy$$

$$= 2\left[y+\frac{y\sqrt{1-y^2}}{2}+\tfrac{1}{2}\sin^{-1}y-\frac{y\sqrt{1+y^2}}{2}-\tfrac{1}{2}\ln\left(y+\sqrt{1+y^2}\right)\right]_0^{\sqrt{\sqrt{3}/2}}$$

$$= 2\left[\sqrt{\sqrt{3}/2}+\frac{\sqrt{\sqrt{3}/2}\sqrt{1-\sqrt{3}/2}}{2}+\tfrac{1}{2}\sin^{-1}\left(\sqrt{\sqrt{3}/2}\right)-\frac{\sqrt{\sqrt{3}/2}\sqrt{1+\sqrt{3}/2}}{2}-\tfrac{1}{2}\ln\left(\sqrt{\sqrt{3}/2}+\sqrt{1+\sqrt{3}/2}\right)\right]$$

= 1.2952 (approx)