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Yahoo answers 15-12-2013

Problem: Consider a Circle with center C and diameter AB = 2r. Two parallel chords DE and XY which are equidistant from the center which makes an acute angle θ with the diameter AB and intersect AB at P and Q respectively. The distances CP and CQ are 'a'. Find the area enclosed between the chords.

Solution:

Case (i): Let us find the area bounded by the chords DE and XY when they are perpendicular to the diameter AB.

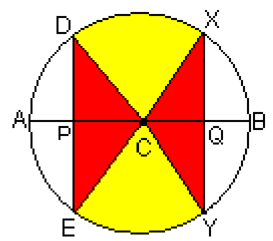

Fig-1

In Fig-1, AB = 2r, CP=CQ=a , DE and XY are perpendicular to AB.

Now $\Delta CXY = \Delta CDE = \frac{1}{2}(XY)(CQ) = \frac{1}{2}(CQ)(2XQ) = a\sqrt{r^2 - a^2}$

$\angle XCD = \angle CXQ + \angle CYQ = 2\angle CXQ = 2\sin^{-1}\left(\frac{a}{r}\right)$ in radians.

Area of sector XCD = Area of sector ECY = $\left(\frac{1}{2}\right)r^2 \angle XCD = \left(\frac{1}{2}\right)r^2 2\sin^{-1}\left(\frac{a}{r}\right) = r^2\sin^{-1}\left(\frac{a}{r}\right)$

Area bounded by the chords DE and XY = 2(ΔCXY + Area of sector XCD)

= $2\left(a\sqrt{r^2 - a^2} + r^2\sin^{-1}\left(\frac{a}{r}\right)\right)$.

Thus, the area bounded by any two parallel chords of a circle of radius 'r' which are at a distance of 'a' from the centre is $2\left(a\sqrt{r^2 - a^2} + r^2\sin^{-1}\left(\frac{a}{r}\right)\right)$. ------------(1)

Case (ii): Let us find the area bounded by the chords DE and XY when they cut the fixed diameter AB at P and Q respectively at an angle θ with the diameter AB.

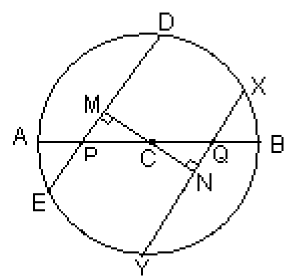

Fig-2

In Fig-2, AB = 2r, CP=CQ=a , $\angle DPB = \angle XQB = \theta$.

Draw MN passes through C and perpendicular to DE and XY.

In the right angled triangles CMP and CNQ, $CM = CN = a\sin\theta$

Now the chords DE and XY are at a distance $CM = CN = a\sin\theta$ from the center C. By case(i), the area bounded by the chords DE and XY is

$2\left(a\sin\theta\sqrt{r^2 - (a\sin\theta)^2} + r^2\sin^{-1}\left(\frac{a\sin\theta}{r}\right)\right)$ (Replace 'a' by '$a\sin\theta$' in (1))