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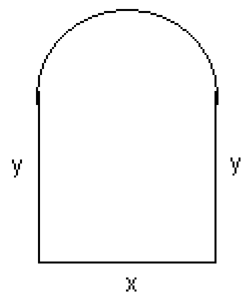
Problem: Suppose that 900 ft of fencing are used to enclose a corral in the shape of a rectangle with a semicircle whose diameter is a side of the rectangle as in the figure below. Find the dimensions of the corral with maximum area.(Find x and y).

Solution: Let x ft be the diameter of the semi-circle (and hence one side of the rectangle) and y ft be the other side of the rectangle.

Area = $xy + \frac{1}{2}\pi (x/2)^2 = xy + \pi x^2/8$.



Perimeter = $x + 2y + \pi x/2 = 900$

Let $f(x,y) = xy + \pi x^2/8$ and $g(x,y) = x + 2y + \pi x/2 - 900 = 0$

Let us apply, Lagrange's multiplier method.

The Lagrange's function $L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$

$L = xy + \pi x^2/8 + \lambda(x + 2y + \pi x/2 - 900)$

$\partial L/\partial x = 0 \Rightarrow y + \pi x/4 + \lambda (1 + \pi/2) = 0$ --------------(1)

$\partial L/\partial y = 0 \Rightarrow x + \lambda (2) = 0$ ---------------(2)

$\partial L/\partial \lambda = 0 \Rightarrow x + 2y + \pi x/2 - 900 = 0$ ---------------(3)

From (2), we have $\lambda = -x/2$.

Put $\lambda = -x/2$ in (1), we get $y = x/2$.

Put $y = x/2$ in (3), we get

$x + 2(x/2) + \pi x/2 - 900 = 0$

⇨ $x(2 + \pi/2) = 900$

⇨ $x = 1800/(4 + \pi) = 252$ and hence $y = 126$.

We get the maximum area when x = 252 and y = 126.