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Problem: A company manufactures cylindrical paint cans with open tops with a volume of 35,000 cubic centimeters. What should be the dimensions of the cans in order to use the least amount of metal in their production?

Solution: Let r be the base radius and h be the height of the open cylinder of least surface area.

Given, Volume = $\pi r^2 h = 35000$.

Surface area of the open cylinder = $\pi r^2 + 2\pi rh$

Let $f(r,h) = \pi r^2 + 2\pi rh$ and $g(r,h) = \pi r^2 h - 35000 = 0$

Let us apply, Lagrange's multiplier method.

The Lagrange's function $L(r,h,\lambda) = f(r,h) + \lambda\, g(r,h) = \pi r^2 + 2\pi rh + \lambda(\pi r^2 h - 35000)$

$\partial L/\partial r = 0 \Rightarrow 2\pi r + 2\pi h + \lambda(2\pi rh) = 0$ ----------(1)

$\partial L/\partial h = 0 \Rightarrow 2\pi r + \lambda(\pi r^2) = 0$ ----------(2)

$\partial L/\partial \lambda = 0 \Rightarrow \pi r^2 h - 35000 = 0$ ----------(3)

From (1) and (2), we have, $(2\pi r + 2\pi h)/(2\pi rh) = (2\pi r)/(\pi r^2) = -\lambda$

- $(r+h)/rh = 2/r$
- $h = r$

Put $h = r$ in (3), we get $r^3 = 35000/\pi$

$h = r = \text{cuberoot}(35000/\pi) = 22.33$ cm