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Problem: A box with a square base and open top must have a volume of 32,000 cm3. Find the dimensions of the box that minimize the amount of material used.
Solution: Let x cm be the side of the base square and y be the height of the box.

Volume of the box = $x^2y = 32000$

Surface area of the box = $x^2+4xy = x^2+4x(32000/x^2)= x^2+k/x$ where k = 128000.

Let $f(x) = x^2+k/x$

$df/dx = 2x – k/x^2$ and $d^2f/dx^2 = 2 + 2k/x^3$

f(x) is maximum or minimum when df/dx = 0

- ⇨ $2x – k/x^2 = 0$
- ⇨ $x^3 = k/2 = 64000$
- ⇨ $x = 40$

when x = 40, $d^2f/dx^2 = 2 + 2k/40^3 = 6 > 0$

Thus f(x) is minimum when x = 40.

x = 40 => $y = 32000/40^2 = 20$.

For minimum material, the dimensions of the box : (40,40,20).