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Problem: What is the shape of the cheapest rectangular box of given volume V_0 if the base material costs twice as much as the material used to make the top and the sides?

Solution: Let the required box size be (x,y,z) units and let 2K be the material cost per square unit for the bottom side.

Volume of the box = $xyz = V_0$

Cost of the box $f(x,y,z) = 2Kxy + K(xy + 2xz + 2yz)$

Let $g(x,y,z) = xyz - V_0 = 0$

Let us solve the problem by Lagrange's multiplier method.

$L(x,y,z, \lambda) = f(x,y,z) + \lambda\, g(x,y,z) = 2Kxy + K(xy + 2xz + 2yz) + \lambda(xyz - V_0)$

$\partial L/\partial x = 2Ky+K(y+2z)+ \lambda yz = 0 \Rightarrow 3Ky+2Kz + \lambda yz = 0$ --------(1)

$\partial L/ \partial y = 2Kx+K(x+2z)+ \lambda xz = 0 \Rightarrow 3Kx + 2Kz + \lambda xz = 0$ ----------(2)

$\partial L/ \partial z = K(2x+2y)+ \lambda xy = 0 \Rightarrow 2Kx + 2Ky + \lambda xy = 0$ ---------(3)

$\partial L/\partial \lambda = xyz - V_0 = 0$ ---------(4)

From (1), (2) and (3), we get

$3/z + 2/y = 3/z + 2/x = 2/y + 2/x = -\lambda/K$

⇨ $z = (3/2)x = (3/2)y$ and $x = y$

From (4), we have $z^3 = 9V_0/4 \Rightarrow z = (9V_0/4)^{(1/3)}$

Hence $x = y = (2V_0/3)^{(1/3)}$

Shape of the rectangular box is $((2V_0/3)^{(1/3)}, (2V_0/3)^{(1/3)}, (9V_0/4)^{(1/3)})$