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Question: If m is a positive integer, show that $\int\limits_0^{\frac{\pi}{2}} \cos^m x \sin^m x dx = 2^{-m} \int\limits_0^{\frac{\pi}{2}} \cos^m x dx$

Solution: $\int\limits_0^{\frac{\pi}{2}} \cos^m x \sin^m x dx = 2^{-m} \int\limits_0^{\frac{\pi}{2}} (2 \sin x \cos x)^m dx = 2^{-m} \int\limits_0^{\frac{\pi}{2}} (\sin 2x)^m dx$

$= 2^{-m} \int\limits_0^{\pi} (\sin u)^m \frac{du}{2}$ (put $2x = u, dx = \frac{du}{2}$)

$= 2^{-m} \left(\frac{1}{2}\right) 2 \int\limits_0^{\frac{\pi}{2}} (\sin u)^m du$ (By the property: $\int\limits_0^{2a} f(x)dx = 2 \int\limits_0^{a} f(x)dx$ if $f(2a - x) = f(x)$)

$= 2^{-m} \int\limits_0^{\frac{\pi}{2}} (\sin(\frac{\pi}{2} - u))^m du$ (By the property: $\int\limits_0^{a} f(x)dx = \int\limits_0^{a} f(a - x)dx$)

$= 2^{-m} \int\limits_0^{\frac{\pi}{2}} (\cos u)^m du$

$= 2^{-m} \int\limits_0^{\frac{\pi}{2}} \cos^m x dx$ (Changing the dummy variable u to x)