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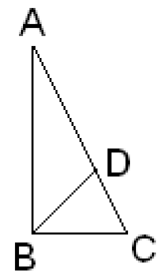
Problem: In a right triangle, the bisector of the right angle divides the hypotenuse in the ratio of 3 to 5. Determine the measures of the acute angles of the triangles

Solution:



let BD be the angle bisector of $\angle B$. Hence $\angle ABD = \angle CBD = 45^\circ$

Let CD = 3k and AD=5k where k is a constant.

Let $\angle BDC = \alpha$ and hence $\angle ADB = 180^\circ - \alpha$.

We have AB = 8k cos A and BC = 8k sin A.

By sine rule in ΔBCD, we have

$CD/\sin 45^\circ = BC/\sin\alpha$

- ⇨ $\sin\alpha = BC\sin 45^\circ/CD = 8k\sin A\,(1/\sqrt{2})/(3k) = 8\sin A/(3\sqrt{2})$ -------(1)

By sine rule in ΔABD, we have

$AD/\sin 45^\circ = AB/\sin(180^\circ - \alpha)$

- ⇨ $\sin(180^\circ - \alpha) = AB\sin 45^\circ/AD = 8k\cos A\,(1/\sqrt{2})/(5k) = 8\cos A/(5\sqrt{2})$
- ⇨ $\sin\alpha = 8\cos A/(5\sqrt{2})$ -----------------------(2)

From (1) and (2), $8\sin A/(3\sqrt{2}) = 8\cos A/(5\sqrt{2})$

- ⇨ $\tan A = 3/5$
- ⇨ $A = \arctan(3/5)$

Hence $B = 90^\circ - \arctan(3/5) = \arctan(5/3)$.

Thus $A = \arctan(3/5)$ and $B = \arctan(5/3)$.