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Problem:

1)find a and b if $4x^4 + ax^3 + bx^2 + 6x + 1 = [P(x)]^2$

2)Find P(5) - P(3) if $P(2x) + P(4x) + P(6x) = 24x - 6$

3)Find the polynomials which satisfy $2P(x) = P(2x)$

4)Find P(x) if $P(x - 2) + P(x + 1) = 6x + 5$

5) Find Q(x) if $Q(x) \times Q(2x) = 8x^2 + 30x + 25$

Solution:

1) Let $P(x) = cx^2+dx+e$

$[P(x)]^2 = c^2x^4 + d^2x^2+e^2+2cdx^3+2dex+2cex^2 = c^2x^4+2cdx^3+(d^2+2ce)x^2+2dex+e^2$

Equating like powers of x, we get

$c^2 = 4$, $2cd=a$, $(d^2+2ce)=b$, $2de=6$, $e^2=1$

$\Rightarrow c = \pm 2$, $e = \pm 1$, $d = \pm 3$

$\Rightarrow (c,d,e) = (2,3,1)$ or $(2,-3,-1)$ or $(-2,3,1)$ or $(-2,-3,-1)$

when $(c,d,e) = (2,3,1)$, $a= 12$ and $b = 13$

when $(c,d,e) = (2,-3,-1)$, $a= -12$ and $b = 5$

when $(c,d,e) =(-2,3,1)$, $a = -12$ and $b = 5$

when $(c,d,e) = (-2,-3,-1)$, $a= 12$ and $b=13$

Thus we have $(a,b) = (12,13)$ or $(-12,5)$.

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2) Let $P(x) = ax+b$

Now $P(2x)+P(4x)+P(6x) = (2ax+b)+(4ax+b)+(6ax+b) = 24x - 6$

$\Rightarrow 12a = 24$ and $3b = -6$

$\Rightarrow a = 2$ and $b = -2$

Hence $P(x) = 2x - 2$

$\Rightarrow P(5) - P(3) = (10 -2) - (6-2) = 8 - 4 = 4$

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3) $P(x) = x$

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4) Let $P(x) = ax+b$

Now $P(x-2)+p(x+1) = a(x-2)+b+a(x+1)+b = 6x+5$

=> $2a = 6$ and $-a+2b = 5$

=> $a=3$ and $b = 4$

Thus $P(x) = 3x+4$

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5) Let $Q(x) = ax+b$, $Q(2x) = 2ax+b$

$Q(x)*Q(2x) = 2a^2x^2+3abx + b^2 = 8x^2+30x+25$

=> $(a,b) = (2,5)$ or $(-2,-5)$.

Thus $Q(x) = 2x+5$ or $-2x-5$