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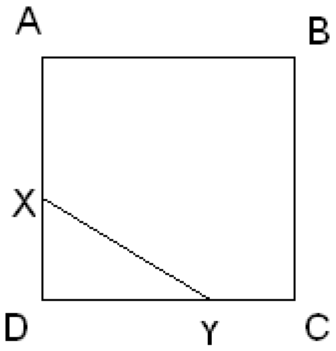
Problem: Take a square piece of paper ABCD and fold the upper left corner A down along the lower edge CD. Call the point where the left edge of the paper is bent point X, and the point along the lower edge where A touches point Y. The triangle XDY is a right triangle in the lower left corner. Turn this situation into an algebraic model to optimize the selection of X so that the area of triangle XDY is maximal.

Solution:



Let the side of the square ABCD be 'a' units.

Let DX = x and hence XY = a-x.

Since XDY is a right triangle, $DY^2 = XY^2 - DX^2 = (a-x)^2 - x^2 = a^2 - 2ax$

$DY = \sqrt{(a^2 - 2ax)}$

Area of the triangle XDY, $f(x) = (1/2)*DX*DY = (1/2)\, x\sqrt{(a^2 - 2ax)}$

$f(x) = (1/2)\sqrt{(a^2x^2 - 2ax^3)}$

$df/dx = (1/2)\,[1/(2\sqrt{(a^2x^2 - 2ax^3)})]*(2a^2x - 6ax^2) = (1/2)\,[(a^2x - 3ax^2)/\sqrt{(a^2x^2 - 2ax^3)}]$

$d^2f/dx^2 = (1/2)[\{\sqrt{(a^2x^2 - 2ax^3)} * (a^2 - 6ax) - (a^2x - 3ax^2) * (a^2x - 3ax^2)/\sqrt{(a^2x^2 - 2ax^3)}\} / (a^2x^2 - 2ax^3)]$

$= (1/2)[\{(a^2x^2 - 2ax^3) * (a^2 - 6ax) - (a^2x - 3ax^2)^2\} / (a^2x^2 - 2ax^3)^{3/2}]$

f(x) is maximum or minimum when df/dx = 0

⇨ $(1/2)\,[(a^2x - 3ax^2)/\sqrt{(a^2x^2 - 2ax^3)}] = 0$

⇨ x = 0 or a/3

x = 0 is not possible.

When x = a/3, $d^2f/dx^2 = -3\sqrt{3} < 0$

Hence f is maximum when x = a/3.

Maximum area of the triangle XDY = $(1/2).(a/3)\sqrt{(a^2 - 2a.a/3)} = a^2/(6\sqrt{3})$