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Yahoo answers 18-11-2013

Problem: Solve $\log_2 x - \log_x y = -1$, $2\log_4 x + \log_4 y = 2$

Solution:

Given $\log \log_2 x - \log_x y = -1$ ----------(1)

$2\log_4 x + \log_4 y = 2$ ----------(2)

Let us simplify (2),

(2) $\Rightarrow$ $\log_4(x^2)+\log_4(y) = 2$ $\Rightarrow$ $\log_4(x^2y) = 2$ $\Rightarrow$ $x^2y = 4^2 = 16$

$\Rightarrow$ $y = 16/x^2$ ----------(3)

substitute (3) in (1), we get

$\log_2(x) - \log_x (16/x^2) = -1$

$\log_2(x) -\{ \log_x(16) - \log_x(x^2)\} = -1$

$\log_2(x) + 2 \log_x(x) - \log_x(2^4) = -1$

$\log_2(x) + 2*1 - 4 \log_x(2) = -1$

$\log_2(x) - 4 \log_x(2) = -3$

put $\log_2(x) = u$ then $\log_x(2) = 1/u$

we have, $u - 4(1/u) = -3$ $\Rightarrow$ $u^2 - 4 = -3u$ $\Rightarrow$ $u^2 + 3u - 4 = 0$

$(u+4)(u-1) = 0$ $\Rightarrow$ $u = -4$ or 1

$\Rightarrow \log_2(x) = -4$ or 1 $\Rightarrow$ $x = 2^{-4}$ or 2^1 $\Rightarrow$ $x = 1/16$ or 2

put $x = 1/16$ in (3), we get $y = 16/(1/16)^2 = 16^3 = 4096$

put $x = 2$ in (3), we get $y = 16/2^2 = 4$

Solution set is **{(1/16, 4096), (2, 4)}**