Dr. K. Karuppasamy

www.drkk.in

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Problem: Use a triple integral to find the volume of the solid enclosed by the paraboloid x=9y^2+9z^2 and the plane x=9. in rectangular coordinates only.

Solution:

In the triple integral x varies from $9(y^2+z^2)$ to 9

The projection of the solid in the yz-plane is a unit circle $y^2+z^2 = 1$.

In this unit circle, y varies from $-\sqrt{1-z^2}$ to $\sqrt{1-z^2}$ and z varies from -1 to 1.

Hence the required volume of the solid = $\int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}\int_{9(y^2+z^2)}^{9} dx\,dy\,dz = \int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}[9-9(y^2+z^2)]\,dy\,dz$

$$= 9\int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}[(1-z^2)-y^2]\,dy\,dz = 18\int_{-1}^{1}\int_{0}^{\sqrt{1-z^2}}[(1-z^2)-y^2]\,dy\,dz = 18$$

$$= 18\int_{-1}^{1}\left[(1-z^2)\sqrt{1-z^2} - \frac{\left(\sqrt{1-z^2}\right)^3}{3}\right]dz = 18\left(\frac{2}{3}\right)2\int_0^1\left(\sqrt{1-z^2}\right)^3 dz$$

$$= 24\int_0^1\left(\sqrt{1-z^2}\right)^3 dz = 24\int_0^{\pi/2}\cos^4\theta\,d\theta \quad \text{(put } z=\sin\theta\text{)}$$

$$= 24\left(\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}\right) = \frac{9\pi}{2}$$