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Problem: Suppose that three "fair" dice are tossed. Determine the probability that the sum of the dice is at most 16.

Solution: Let S(n,k) be the number of cases of the sum being k when n dice are thrown.

$S(n,k)$ = coefficient of x^k in $(x+x^2+x^3+x^4+x^5+x^6)^n$

= coefficient of x^k in $x^n*(1+x+x^2+x^3+x^4+x^5)^n$

= coefficient of x^k in $x^n * [(1-x^6)/(1-x)]^n$

=coefficient of x^k in $x^n * (1-x^6)^n *(1-x)^{-n}$

Now $S(3,17)$ = coefficient of x^{17} in $x^3*(1-x^6)^3*(1-x)^{-3}$

= coefficient of x^{17} in $x^3 *(1 -3x^6+3x^{12}-x^{18})* (1-x)^{-3}$

= coefficient of x^{17} in $(x^3 -3x^9+3x^{15}-x^{21})* (1-x)^{-3}$

= coefficient of x^{17} in $(x^3 -3x^9+3x^{15}-x^{21})*(1+ (2*3)/(1*2)\, x + (3*4)/(1*2)\, x^2+(4*5)/(1*2)\, x^3+ \ldots)$

$= 1* (15*16)/(1*2) + (-3)*(9*10)/(1*2)+3*(3*4)/(1*2) = 3$

Obviously $S(3,18) = 1$ (or)

$S(3,18)$ = coefficient of x^{18} in $(x^3 -3x^9+3x^{15}-x^{21})*(1+ (2*3)/(1*2)\, x + (3*4)/(1*2)\, x^2+(4*5)/(1*2)\, x^3+ \ldots)$

$=1* (16*17)/(1*2) + (-3)*(10*11)/(1*2)+3*(4*5)/(1*2) =1$

Exhaustive number of cases = 6*6*6 = 216

Favorable number of cases for getting the sum as at most 16 = 216 - S(3,17)- S(3,18)= 212

Required Probability = 212/216 =53/54