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Problem: How many triples a, b, c of real numbers are there such that a, b, c are the roots of the equation $x^3 + ax^2 + bx + c = 0$?

Solution: Sum of the roots, $a+b+c = -a \Rightarrow 2a+b+c = 0$ -----(1)

Sum of the roots taken two at a time, $ab+bc+ca = b \Rightarrow c = (b-ab)/(a+b)$ -----(2)

Product of the roots, $abc = -c \Rightarrow ab = -1; b = -1/a$ -----(3)

put $b = -1/a$ in (1), we get $c = (2a^2-1)/a$ -----(4)

put $b = -1/a$ in (2), we get $c = 1/(a+1)$ -----(5)

Equating (4) and (5), we get $2a^3 + 2a^2 - 2a - 1 = 0$.

Solving this cubic equation, we get $a = -1.4516, -0.40303, 0.85464$.

Corresponding b 's are $0.6889, 2.4812, -1.1701$

Corresponding c 's are $-2.2143, 1.6751, 0.5392$

Thus we have three triplets, satisfying the required condition.