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Question: A fifth degree polynomial $f(x)$ has the leading coefficient 252 and $f(2) = 3$, $f(3) = 8, f(4) = 15, f(5) = 24, f(6) = 35$ but $f(1) \neq 0$, then find $f(1)/252$.

Solution: Let $f(x) = 252(x-2)(x-3)(x-4)(x-5)(x-6) + a_1(x-2)(x-3)(x-4)(x-5) + a_2(x-2)(x-3)(x-4) + a_3(x-2)(x-3) + a_4(x-2) + a_5$.
$f(2) = 3 \Longrightarrow a_5 = 3$
$f(3) = 8 => a_4 = 5$
$f(4) = 15 => a_3 = 1$
$f(5) = 24 => a_2 = 0$
$f(6) = 35 => a_1 = 0$
Hence $f(x) = 252(x-2)(x-3)(x-4)(x-5)(x-6) + (x-2)(x-3) + 5(x-2) + 3$
Now $f(1) = -30240$
Thus $f(1)/252 = -120$.