Dr. K. Karuppasamy

www.drkk.in

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Problem : A fair die is rolled once, and the number score is noted. Let the random variable X be twice this score, and define the variable Y to be one if an odd number appeared and three if an even number arose. By finding the probability mass function in each case, find the expectation of the following random variables: a) X b) Y c) XY .

Solution: The joint probability mass function of (X,Y) is

		X						Total
		2	4	6	8	10	12	
Y	1	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{2}$
	3	0	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$	$\frac{1}{2}$
Total		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

The marginal probability mass function of X is

x	2	4	6	8	10	12
$P_X(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

The marginal probability mass function of Y is

y	1	3
$P_Y(y)$	$\frac{1}{2}$	$\frac{1}{2}$

1) $E(X) = \sum xP_X(x) = 2\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) = 7$

2) $E(Y) = \sum yP_Y(y) = 1\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) = 2$

3) $E(XY) = \sum\sum xyP(x, y)$

$= (2)(1)\left(\frac{1}{6}\right) + (6)(1)\left(\frac{1}{6}\right) + (10)(1)\left(\frac{1}{6}\right) + (4)(3)\left(\frac{1}{6}\right) + (8)(3)\left(\frac{1}{6}\right) + (12)(3)\left(\frac{1}{6}\right)$

= 15