Dr. K. Karuppasamy

www.drkk.in

## Yahoo answers 25-09-2013

**Problem**: A wire of length b is cut in two parts which are bent in the form of square and circle respectively. Find the least value of the sum of areas so formed.

**Solution**: Let x and y be the two parts, where x is converted into a square and y is converted into a circle. Let a be the side of square and r be the radius of circle so formed. Thus we have,

$$4a = x\ ,\ 2\pi r = y \text{ and } x+y = b.$$

- ⇨ $a = x/4$ and $r = y / (2\pi)$ and $x + y = b$

  Let $f(x,y)$ = sum of the areas of square and circle = $(x/4)^2 + \pi (y/2\pi)^2$ and let $g(x,y)=x+y-b=0$.

  The auxiliary function be $F(x,y) = f(x,y)+\lambda g(x,y) =(x/4)^2 + \pi (y/2\pi)^2 +\lambda(x+y-b)$

  $\partial F/\partial x = 0 \quad => x/8 + \lambda= 0$ ----------(1)

  $\partial F/\partial y = 0 \quad => y/(2\pi) + \lambda= 0$ ----------(2)

  $\partial F/\partial \lambda = 0 \quad => x+y-b=0$ ----------(3)

  From (1) and (2), $-\lambda= x/8 = y/(2\pi)$

- ⇨ $x/8 = y/(2\pi) = (x+y)/(8+2\pi) = b/(8+2\pi)$ (using (3))
- ⇨ $x = (8b)/(8+2\pi) = (4b)/(4+\pi)$ and $y= (2\pi b)/(8+2\pi) = (\pi b)/(4+\pi)$

  Least value of sum of areas = $(x/4)^2 + \pi (y/2\pi)^2 =$ $\mathbf{[b/(4+\pi)]^2+\pi[b/(8+2\pi)]^2}$