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Yahoo answers 01-09-2013

Problem: How many numbers from 1 to 1,000,000 having its digit sum equal to 9?

Solution: Partition the number 9 as a sum of 6 non-negative integers between 0 and 9.

i.e., $9 = a+b+c+d+e+f$. The number of such different partitions of 9 is the required answer. To avoid repetitions, assume $0 \leq a \leq b \leq c \leq d \leq e \leq f \leq 9$.

Recall that the number of permutations of the digits of aabbccccc is $\frac{9!}{2!3!4!}$.

The possible partitions of 9 are listed below with its digits permutation.

Sno	Partition	Permutations
1.	(0,0,0,0,9)	6
2.	(0,0,0,0,1,8)	30
3.	(0,0,0,0,2,7)	30
4.	(0,0,0,0,3,6)	30
5.	(0,0,0,0,4,5)	30
6.	(0,0,0,1,1,7)	60
7.	(0,0,0,1,2,6)	120
8.	(0,0,0,1,3,5)	120
9.	(0,0,0,1,4,4)	60
10.	(0,0,0,2,2,5)	60
11.	(0,0,0,2,3,4)	120
12.	(0,0,0,3,3,3)	20
13.	(0,0,1,1,1,6)	60
14.	(0,0,1,1,2,5)	180
15.	(0,0,1,1,3,4)	180
16.	(0,0,1,2,2,4)	180
17.	(0,0,1,2,3,3)	180
18.	(0,0,2,2,2,3)	60
19.	(0,1,1,1,1,5)	30
20.	(0,1,1,1,2,4)	120
21.	(0,1,1,1,3,3)	60
22.	(0,1,1,2,2,3)	180
23.	(0,1,2,2,2,2)	30
24.	(1,1,1,1,1,4)	6
25.	(1,1,1,1,2,3)	30
26.	(1,1,1,2,2,2)	20
	Total	2002

Required answer is 2002.