

Yahoo Answer dated 24-09-2013

Question: Solve the following initial value problem $(\sin^2 y + x \cot y) \frac{dy}{dx} = 1, x(\frac{\pi}{4}) = \frac{1}{2}$.

Solution: $\frac{dy}{dx} = \frac{1}{\sin^2 y + x \cot y}$

$$\implies \frac{dx}{dy} = \sin^2 y + x \cot y$$

$$\implies \frac{dx}{dy} - x \cot y = \sin^2 y$$

Here $P = -\cot y, Q = \sin^2 y$

Integrating factor, $I.F. = e^{\int P dy} = e^{-\int \cot y dy} = e^{-\ln(\sin y)} = \frac{1}{\sin y}$

General solution, $x(I.F.) = \int Q(I.F.) dy + C$

$$\implies \frac{x}{\sin y} = \int \sin^2 y \frac{1}{\sin y} dy + C$$

$$\implies \frac{x}{\sin y} = \int \sin y dy + C$$

$$\implies \frac{x}{\sin y} = -\cos y + C$$

$$\implies x = -\sin y \cos y + C \sin y$$

Now $x(\frac{\pi}{4}) = \frac{1}{2}$ implies, $\frac{1}{2} = -\sin(\frac{\pi}{4}) \cos(\frac{\pi}{4}) + C \sin(\frac{\pi}{4})$

$$\implies \frac{1}{2} = -\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + C \frac{1}{\sqrt{2}}$$

$$\implies C = \sqrt{2}$$

Particular solution is, $x = -\sin y \cos y + \sqrt{2} \sin y = \sin y(\sqrt{2} - \cos y)$