

Yahoo Answer dated 23-09-2013

Question: Using Lagrange multiplier to maximize $f(x, y, z) = xyz$, with the restriction $x + y + z = 3$, given that $x > 0; y > 0; z > 0$.

Solution: Let the auxiliary function be $F(x, y, z) = xyz + \lambda(x + y + z - 3)$.

$$\frac{\partial F}{\partial x} = 0 \implies yz + \lambda = 0$$

$$\frac{\partial F}{\partial y} = 0 \implies xz + \lambda = 0$$

$$\frac{\partial F}{\partial z} = 0 \implies xy + \lambda = 0$$

$$\frac{\partial F}{\partial \lambda} = 0 \implies x + y + z - 3 = 0$$

From first three equations, we have $yz = xz = xy = -\lambda$

$$\implies yz = xz = xy \implies \frac{1}{x} = \frac{1}{y} = \frac{1}{z} = \frac{1+1+1}{x+y+z} = \frac{3}{3} = 1$$

$\implies x = y = z = 1$. Maximum of $f(x, y, z) = 1 * 1 * 1 = 1$.