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Yahoo Answer dated 18-09-2013

Question: Gind that $\cosh x = \frac{8}{7}$, determine the value of $\sinh x$ and $\tanh x$.

Solution: Given that $\cosh x = \frac{8}{7}$

$\implies \frac{e^x + e^{-x}}{2} = \frac{8}{7} \implies e^x + e^{-x} = \frac{16}{7}$

$\implies e^{2x} + 1 = \frac{16 e^x}{7} \implies 7e^{2x} - 16e^x + 7 = 0$

Solving this quadratic equation in e^x, we get $e^x = \frac{8 \pm \sqrt{15}}{7}$.

Also $e^{-x} = \frac{7}{8 \pm \sqrt{15}} = \frac{8 \mp \sqrt{15}}{7}$

Now $\sinh x = \frac{e^x - e^{-x}}{2} = \frac{\sqrt{15}}{7}$ or $-\frac{\sqrt{15}}{7}$

and hence $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\sqrt{15}}{8}$ or $-\frac{\sqrt{15}}{8}$