Dr. K. Karuppasamy

(www.drkk.in)

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Question: In any triangle ABC, prove that $(a-b)^2\cos^2(c/2)+(a+b)^2\sin^2(c/2)=c^2$?

Solution: LHS $=(a-b)^2\cos^2(c/2)+(a+b)^2\sin^2(c/2)=(a-b)^2(1-\sin^2(c/2))+(a+b)^2\sin^2(c/2)$

$=\left\{(a+b)^2-(a-b)^2\right\}\sin^2(c/2)+(a-b)^2=4ab\left(\frac{1-\cos C}{2}\right)+(a-b)^2$

$=2ab-2ab\cos C+\left(a^2+b^2-2ab\right)=a^2+b^2-2ab\cos C=c^2=$ RHS.