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Problem: Let $L = \{(x, y) \mid -2x + y = 3\}$ contained in R^2. Given any point (c,d) in R^2, find a point on L which is closest to (c,d).

Solution: Let the required point be (x,y), Therefore, y=2x+3.

Now the distance between (x,y) and (c,d) is $D = \sqrt{(x-c)^2 + (y-d)^2}$.

Let $f(x) = D^2 = (x-c)^2 + (2x+3-d)^2$, $f'(x) = 2(x-c) + 4(2x+3-d)$ and $f''(x) = 10$.

$f(x)$ is minimum when $f'(x) = 2(x-c) + 4(2x+3-d) = 0 \Rightarrow x = \frac{c+2d-6}{5}$ and hence $y = 2\left(\frac{c+2d-6}{5}\right) + 3 = \frac{2c+4d+3}{5}$. Also, at this point $f''(x) = 10 > 0$.

Hence the required point on L is $\left(\frac{c+2d-6}{5}, \frac{2c+4d+3}{5}\right)$.