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Yahoo Answer dated 13-09-2013

Question: Find the area bounded by the parabola $y = \frac{x^2}{2}$ and the hyperbola $y^2 - x^2 = 8$.

Solution: Put $x^2 = 2y$ in $y^2 - x^2 = 8$, we get $y = 4, -2$.
Since $y > 0$, put $y = 4$ in $y = \frac{x^2}{2}$, we get $x = \pm\sqrt{8}$.
Required area $= \int_{-\sqrt{8}}^{\sqrt{8}} \left(\sqrt{8+x^2} - \frac{x^2}{2}\right)\ dx = 2\int_0^{\sqrt{8}} \left(\sqrt{8+x^2} - \frac{x^2}{2}\right)\ dx$

$= 2\left[\frac{x\sqrt{8+x^2}}{2} + 4\sinh^{-1}\frac{x}{\sqrt{8}} - \frac{x^3}{6}\right]_0^{\sqrt{8}} = 2\left[\frac{\sqrt{8}\sqrt{16}}{2} + 4\sinh^{-1}1 - \frac{\sqrt{8}^3}{6}\right]$

$= 8\sqrt{2} + 8\sinh^{-1}1 - \frac{16\sqrt{2}}{3}$