Kalasalingam University

Department of Mathematics

Question Bank for Mathematics III (MAT202)

Unit I : Partial Differential Equation

Part – A

1. Form the partial differential differential equation by eliminating arbitrary constants from $z = ax + by + \sqrt{a^2 + b^2}$.
2. Form the pde by eliminating the arbitrary constants from $(x-a)^2 + (y-b)^2 + z^2 = 1$.
3. Form the pde by eliminating the arbitrary constants from $\log(az-1) = x + ay + b$.
4. Form the pde by eliminating the arbitrary constants from $z = (x^2 + a)(y^2 + b)$.
5. Find the differential equation of all spheres whose centre s on the z-axis.
6. Obtain the pde by eliminating the function from $z = xy + f(x^2 + y^2 + z^2)$
7. Obtain the pde by eliminating the function from $xyz = \phi(x^2 + y^2 - z^2)$
8. Form pde by eliminating the arbitrary functions f and g in $z = x^2 f(y) + y^2 g(x)$
9. Solve $pq = 1$
10. Solve $p^2 + q^2 = 4$
11. Solve $\sqrt{p} + \sqrt{q} = 1$
12. solve $pq = 4$
13. solve $p^2 + q^2 = x + y$
14. Solve $px + qy = z$
15. solve ($(D^2 - 6DD' + 9D'^2)Z = 0$
16. Solve ($(D^4 - D'^4)z = 0$
17. Find the particular integral of $(D^3 - 3D^2D' + 4D'^3)Z = e^{x+2y}$
18. Find particular integral of $(D^4 - 2D^3D' + 2DD'^3 - D'^4)Z = e^{2x+3y}$
19. Solve $\left(2D^2 + 5DD' + 2D'^2\right)z = 0$
20. Find particular integral of $\left(D^2 - 2DD'\right)z = e^{2x}$

Part-B

1. Form the pde by eliminating the arbitrary constants from $z = f(x + 3y) + g(x - 2y)$
2. Form the pde by eliminating the arbitrary constants from $z = f(x^3 + 2y) + g(x^3 - 2y)$
3. Solve (i). $p^2 + q^2 = 4$ (ii). $p^2 + q^2 = npq$.

4. Solve $z = px + qy + p^2 + pq + q^2$ (ii) $(1-x)p + (2-y)q = 3-z$
5. Solve (i). $z = px + qy - 2\sqrt{pq}$ (ii). $z = px + qy + \sqrt{1+p^2+q^2}$
6. Solve $z^2 = p^2 + q^2 + 1$
7. Solve $z^2 = p^2 + qz$
8. Solve $9(p^2 z + q^2) = 4$
9. Solve (i) $pz = 1 + q^2$ (ii). $z = p^2 + q^2$
10. Solve $z^2(p^2 + q^2 + 1) = 1$
11. Solve (i) $q = px + p^2$ (ii) $p + q = \sin x + \sin y$
12. Solve (i) $p^2 + q^2 = x^2 + y^2$ (ii) $q = xyp^2$
13. Solve (i) $yp = 2yx + \log q$ (ii) $q^2 - p = y - x$
14. Solve $\frac{y^2 z}{x} p + xzq = y^2$
15. Solve $(mz - ny)p + (nx - lz)q = ly - mx$
16. Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$
17. Solve $(3z - 4y)p + (4z - 2z)q = 2y - 3x$
18. Solve $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$
19. Solve $(y+z)p + (z+x)q = x + y$
20. Solve $(y^2 + z^2)p - xyq + xz = 0$
21. Solve ($D^3 + D^2 D' - DD'^2 - D'^3)z = e^{2x+y}$
22. Solve ($D^2 - 2DD')z = e^{2x} + x^3 y$
23. Solve ($D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + x^2 y$
24. Solve $r + s - 6t = y \cos x$.
25. Solve $(D^2 - DD')z = \sin x \cos 2y$

Unit II : Laplace Transform

PART-A

1. Find $L[(1+t)^2]$
2. Find $L[e^{-3t}]$
3. Find $L[e^{2t}]$
4. Find $L[e^{-5it}]$
5. Find $L[e^{4it}]$
6. Find $L[sinh2t]$
7. Find $L[cosh3t]$

8. Find $L[sin5t]$
9. Find $L[cos4t]$
10. Find $L[sin^3 2t]$
11. Find $L[cos^2 3t]$
12. Find $L[e^{2t} sin3t]$
13. Find $L[e^{-3t} cos2t]$
14. Find $L[tsin5t]$
15. Find L[$\frac{sin2t}{t}$]
16. Find L[$\int_0^t \frac{\sin t}{t} dt$]
17. Find 1*sin2t
18. Find the inverse Laplace transform of $\frac{1}{s} - \frac{2}{s-3}$
19. Find the inverse Laplace transform of $\frac{1}{s^2 - 4s - 20}$
20. By Partial fraction find the inverse Laplace Transform of $\frac{1}{s^2 - 3s - 10}$

PART-B

1. Find (i) $L[t\cos^3 t]$ (ii) $L\left[\int_0^t \frac{e^{-t}\sin t}{t} dt\right]$

2. Find the Laplace transform of $\frac{e^{-3t} - e^{-4t}}{t}$ and $e^{-t}\int_0^t t\cos t dt$.

3. Find L[f(t) if $f(t) = \begin{cases} \cos t & when \quad 0 < t < \pi \\ \sin t & when \quad t > \pi \end{cases}$

4. Find $L[f(t)]$ if $f(t) = \begin{cases} t & 0 < t < \pi \\ 2\pi - t & \pi < t < 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$.

5. Find L[f(t) if $f(t) = \begin{cases} \sin \omega t & when \quad 0 < t < \frac{\pi}{\omega} \\ \cos \omega t & when \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ with $f(t + \frac{2\pi}{\omega}) = f(t)$

6. Find the Laplace transform of the rectangular wave given by

$f(t) = \begin{cases} 1, & 0 < t < b \\ -1, & b < t < 2b \end{cases}$ with f(t) = f(t+2b)

7. Find the inverse Laplace transform of $\log\left[\frac{s(s+1)}{s^2+1}\right]$.

8. Find the inverse Laplace transform of $\tan^{-1}\left(\frac{a}{s}\right)$.

9. Find the inverse Laplace transform of $\cot^{-1}(s+1)$.

10. Find $L^{-1}\left[\log\frac{s^2+a^2}{s^2-b^2}\right]$.

11. Find the Laplace transform of $s\log\left(\frac{s-1}{s+1}\right)$.

12. Find $L^{-1}\left[\frac{1}{s(s+1)(s+2)}\right]$.

13. Find $L^{-1}\left[\frac{1-s}{(s+1)(s^2+4s+13)}\right]$.

14. Using convolution theorem, find $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$.

15. Using Laplace Transform, evaluate $\int_0^{\infty}\frac{e^{-t}-e^{-2t}}{t}dt$.

16. Using Laplace Transform, evaluate $\int_0^{\infty} te^{-3t}\sin t\, dt$.

17. Using Laplace Transform, solve $\frac{d^2x}{dt^2}+t\frac{dx}{dt}-5y=5$ given that x=0, $\frac{dx}{dt}=2$ when t = 0.

18. Using Laplace Transform, solve $y''-3y'+2y=e^{2t}, \quad y(0)=-3, y'(0)=5.$

19. Using Laplace Transform, solve $(D^2+D)=t^2+2t$, where y(0) = 4, y'(0) = -2

20. Using Laplace Transform, solve $\frac{dy}{dt}-y=1-2t$, given that y(0) = -1

Unit III : Fourier Series

Part - A

1. State Dirichlet's conditions for Fourier series.
2. Find b_n in the expansion of x^2 as a Fourier Series in $(-\pi, \pi)$.
3. If f(x) is an odd function defined in (-l, l) what are the values of a_0 and a_n?
4. Find the Fourier constant b_n for xsinx in $(-\pi, \pi)$.

5. State Parseval's identity for the half-range cosine expansion of f(x) in (0,1).

6. Find the constant term in the Fourier series expansion of f(x) = x in ($-\pi, \pi$).

7. What is the constant term in the Fourier series expansion of f(x) = $x - x^3$ in (-7,7)?

8. Find the constant term in the Fourier series of f(x) = $\cos^2 x$ in the interval ($-\pi, \pi$).

9. To which value, the half range sine series corresponding to f(x) = x^2 expressed in the interval (0,2) converges at x=2.

10. Find the constant a_0 of the Fourier series for the function f(x) = x in $0 \le x \le 2\pi$.

11. What is the sum of the Fourier series at a point x = x_0 where the function f(x) has a finite discontinuity.

12. Obtain the sine series for unity in (0, π).

13. Define Root Mean Square value of a function.

14. Find the constant a_0 of the Fourier series for the function f(x) = k, $0 < x < 2\pi$.

15. Write the Fourier series in complex form for f(x) defined in the interval c to $c+2\pi$.

16. Find the RMS value of the function f(x) = x in (0,l).

17. Find the value of a_n in the cosine series expansion of f(x) = k in (0,10).

18. To what value, the Fourier series corresponding to f(x) = x^2 in ($0, 2\pi$) converges at x=0.

19. What is the coefficient b_n half-range sine series in the interval 0< x < l.

20. If the Fourier series of the function f(x) = $x + x^2$ in $-\pi < x < \pi$ is $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left(\frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right)$ then find the value of the infinite series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$

Part – B

1. Find the Fourier Series for the function $f(x) = x^2 \ \ in \ \ (0, 2\pi)$.
2. Find the Fourier Series for the function f(x) = e^x in($-\pi$, π).
3. Find the Fourier Series for the function $f(x) = (\pi - x)^2$ in the interval (0, 2π).
4. Find the Fourier Series for the function f(x)=xsinx in in (0, 2π).
5. Find the Fourier Series for the function $f(x) = \begin{cases} x & in \quad 0 \le x \le \pi \\ 2\pi - x & in \quad \pi \le x \le 2\pi \end{cases}$

6. Find the Fourier Series for the function $f(x)=\begin{cases} -\pi & in\ -\pi<x<0 \\ x & in\ \ 0<x<\pi \end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....=\frac{\pi^2}{8}$.
7. Obtain the Fourier Series to represent the following function $f(x)=x^2\ \ in\ \ (-\pi,\pi)$. Hence deduce the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....$
8. Obtain the Fourier Series to represent the following function f(x) = |x| in (-π, π).
9. Obtain the Fourier Series to represent the following function $f(x)=x^2\ \ in\ \ (-\pi,\pi)$. Hence deduce the sum of the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....$
10. Obtain the Fourier Series to represent the following function $f(x)=\begin{cases} \pi+2x & in\ -\pi<x<0 \\ \pi-2x & in\ \ 0<x<\pi \end{cases}$
11. Find the Fourier Series to represent the following function $f(x)=(l-x)^2\ \ in\ \ (0,2l)$.
12. Find the Fourier Series to represent the following function $f(x)=\begin{cases} 1 & in\ \ 0<x<1 \\ x & in\ \ 1<x<2 \end{cases}$
13. Find the Fourier Series to represent the following function f(x) = x^2 for (-l, l).
14. Find the half range Fourier Sine Series to represent $f(x)=\cos x\ \ in\ \ (0,\pi)$
15. Find the half range Fourier Sine Series to represent $f(x)=x^3\ \ in\ \ (0,l)$
16. Find the half range Fourier Cosine Series to represent $f(x)=\sin x\ \ in\ \ (0,\pi)$
17. Find the Complex Form of Fourier Series to represent f(x) = e^{ax} in (–π, π)
18. Find the Fouriere cosine Series of f(x) = x(π-x) in (0, π). Hence show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+....=\frac{\pi^4}{90}$
19. Find the Fourier Series, upto second harmonic, for the function y = f(x) from the following table:

X	0	$\frac{\pi}{3}$	$\frac{\pi}{3}$	π	$\frac{\pi}{3}$	$\frac{\pi}{3}$	2π
y = f(x)	10	12	15	20	17	11	10

20. From the following table, find the Fourier Series to represent y = f(x) upto two harmonics in (0, 6).

X	0	1	2	3	4	5
y = f(x)	9	18	24	28	26	20

Unit IV : Z-Transform

Part A

1. Find Z[a^n]

2. Find Z[n]
3. Find Z[na^n]
4. Find Z[na^{n-1}]
5. Find Z$\left[\frac{1}{n!}\right]$
6. FindZ$\left[\frac{1}{n+1}\right]$
7. FindZ$\left[\frac{1}{n}\right]$
8. Find Z[e^{iat}]
9. Prove that $Z(a^n f(n)) = F\left(\frac{z}{a}\right)$ where $Z(f(n)) = F(z)$.
10. Prove that $Z(f(n-m)) = z^{-m}F(z)$.
11. Find Z$\left[\frac{(n+2)(n+1)}{2}\right]$
12. Find Z$\left[\cos\frac{n\pi}{2}\right]$
13. Z[$e^{-t}t^2$]
14. Z$\left[\frac{a^n}{n!}\right]$
15. Z$\left[4.3^n + 2(-1)^n\right]$
16. Define Z-transform of unit step sequence.
17. State convolution theorem on Z-transform
18. Find Z$\left[\left(\frac{1}{2}\right)^n * \cos n\pi\right]$
19. Find $Z^{-1}\left[\frac{z}{z-2} + e^{\frac{1}{z}}\right]$
20. State and prove Linearity property of z transforms.

Part B

1. (i). Prove that $Z(t^k) = -Tz\frac{d}{dz}\left(Z(t^{k-1})\right)$. Hence deduce $Z(t), Z(t^2)$.

   (ii) Prove that $Z(nf(n)) = -z\frac{d}{dz}F(z)$ where $Z(f(n)) = F(z)$

2. Prove that $Z(f(n+m)) = z^m\left(F(z) - f(0) - \frac{f(1)}{z} - \frac{f(2)}{z^2} - \frac{f(3)}{z^3} .... - \frac{f(m-1)}{z^{m-1}}\right)$ where $Z(f(n)) = F(z)$.

3. (i) Using Long Division method, find the inverse Z-transform of $\frac{10z}{(z-1)(z-2)}$

(ii) Using partial fraction method, find the inverse Z-transform of $\frac{z}{z^2+7z+10}$

4. (i)Using Residue theorem, find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$

(ii) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$

5. (i) Using Long Division method, find the inverse Z-transform of $\frac{4z}{(z-1)^2}$

(ii) Using Residue theorem, find the inverse Z-transform of $\frac{z^2}{(z-a)(z-b)}$

6. (i) Using partial fraction method, find the inverse Z-transform of

$$\frac{z^3}{(z-1)^2(z-2)}$$

(ii) Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z+a)^2}$

7. Using Residue theorem, find the inverse Z-transform of $\frac{z(z^2-z+2)}{(z+1)(z-1)^2}$

8. Using partial fraction method, find the inverse Z-transform of $\frac{z^2+2z}{z^2+2z+4}$

9. Using Residue theorem, find the inverse Z-transform of $\frac{z(3z^2-6z+4)}{(z-2)^2(z+1)}$

10. (i)Using convolution theorem, find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$

(ii) Find z$\left[\frac{1}{n+1} * \frac{a^n}{n!}\right]$

11. Using Long Division method, find the inverse Z-transform of $\frac{z}{2z^2-3z+1}$

12. Using convolution theorem, find the inverse Z-transform of $\frac{z^2}{(z-1)(z-3)}$

13. (i) Using Residue theorem, find the inverse Z-transform of $\frac{z}{z^2-2z+2}$

(ii)Using partial fraction method, find the inverse Z-transform of

$$\frac{z^2-3z}{(z-5)(z+2)}$$

14. (i) Find the z transform for unit step function and unit impulse function

(ii) Find $Z^{-1}\left[\frac{z^2}{(z-4)(z-3)}\right]$

15. Solve the difference equations $y_{n+2}-4y_{n+1}+4y_n=0$ given that $y_0=1, y_1=0$.

16. Solve the difference equations $y_{n+2}-3y_{n+1}+2y_n=2^n$ given that $y_0=y_1=0$.

17. Solve the difference equations $y_{n+2}+y_n=1$ given that $y_0=y_1=0$.

18. Solve the difference equations $6y_{n+2}-y_{n+1}+y_n=0$ given that $y_0=0, y_1=0$.

19. Solve the difference equations $y_{n+2}+2y_{n+1}+y_n=n$ given that $y_0=0, y_1=0$.

20. Solve the difference equations $y_{n+2}-3y_{n+1}-10y_n=0$ given that $y_0=1, y_1=0$.

UNIT V : Fourier Transform

Part - A

1. State Fourier Integral formula.
2. State Fourier sine integral.
3. State Fourier cosine integral.
4. State (Complex) Fourier transform and its inversion.
5. State Fourier sine transform and its inversion.
6. State Fourier cosine transform and its inversion.
7. Find the Fourier sine transform of $f(x)=e^{-ax}$.
8. Find the Fourier cosine transform of $f(x)=e^{-ax}$.
9. Find the Fourier transform of $f(x)=\begin{cases}1 & for \quad |x|\le a\\ 0 & for \quad |x|>a\end{cases}$.
10. Prove that $F\left(f(x)\cos ax\right)=\frac{1}{2}\left[f(s+a)+f(s-a)\right]$ where $F(f(x))=f(s)$.
11. Prove that $F_S\left(f(x)\cos ax\right)=\frac{1}{2}\left[F_S(s+a)+F_S(s-a)\right]$

12. Prove that $F_C\left(f(x)\cos ax\right)=\frac{1}{2}\left[F_C(a+s)+F_C(a-s)\right]$

13. Prove that $F_S\left(f(x)\sin ax\right)=\frac{1}{2}\left[F_C(s-a)-F_C(s+a)\right]$

14. Prove that $F_C\left(f(x)\sin ax\right)=\frac{1}{2}\left[F_S(a+s)+F_S(a-s)\right]$

15. State Parsevals identity on Fourier transform.

PART - B

1. Express the function $f(x)=\begin{cases}1 & for \quad |x|\le 1\\ 0 & for \quad |x|>1\end{cases}$ as a Fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}d\lambda$ and $\int_0^\infty \frac{\sin\lambda}{\lambda}d\lambda$.

2. Using Fourier sine integral of $f(x)=e^{-ax},(a>0)$, ST $\int_0^\infty \frac{\lambda\sin\lambda x}{\lambda^2+a^2}d\lambda=\frac{\pi}{2}e^{-ax}$.

3. Find the Fourier cosine integral of $f(x)=e^{-ax},(a>0)$. Hence deduce the value of the integral $\int_0^\infty \frac{\cos\lambda x}{1+\lambda^2}d\lambda$.

4. Find the Fourier Transform of $f(x)=\begin{cases}1-x^2 & in \quad |x|\le 1\\ 0 & in \quad |x|>1\end{cases}$. Hence prove that $\int_0^\infty \frac{\sin s-s\cos s}{s^3}\cos\left(\frac{s}{2}\right)ds=\frac{3\pi}{16}$.

5. Show that $e^{-\frac{x^2}{2}}$ is self-reciprocal under the Fourier Transform.

6. Find FST of $f(x)=e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}dx$

7. Find the FCT of $f(x)=x^{n-1}$. Hence show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under FCT.

8. Find the FST of $f(x)=x^{n-1}$. Hence show that $\frac{1}{\sqrt{x}}$ is self-reciprocal under FST.

9. Find the FST of $f(x)=\frac{e^{-ax}}{x}$.

10. Find the FCT of $f(x)=\frac{e^{-ax}}{x}$.

11. Find the FST of $f(x)=\frac{x}{x^2+a^2}$.

12. Find the FCT of $f(x) = e^{-ax} \cos ax$.

13. Find the FCT of $f(x) = e^{-ax} \sin ax$.

14. Find $f(x)$, if its sine transform is $\frac{e^{-as}}{s}$.

15. Using the properties, find the FST and FCT of xe^{-ax}.

16. Using transforms, evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$.

17. Using transforms evaluate $\int_0^\infty \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$

18. Using transforms, evaluate $\int_0^\infty \frac{x^2}{(x^2 + a^2)^2} dx$.

19. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & in \quad |x| \le 1 \\ 0 & in \quad |x| > 1 \end{cases}$. Using Parseval's identity, show that

$$\int_0^\infty \left(\frac{\sin t}{t} \right)^4 dx = \frac{\pi}{3}.$$

20. Find the Fourier transform of $f(x) = \begin{cases} 1 & for \quad |x| \le a \\ 0 & for \quad |x| > a \end{cases}$ and deduce that (i) $\int_0^\infty \frac{\sin t}{t} dx = \frac{\pi}{2}$ and

(ii) $\int_0^\infty \left(\frac{\sin t}{t} \right)^2 dx = \frac{\pi}{2}$